

UNIT 8: NUMERICAL PROPORTIONALITY.

Ratio:

A **ratio** is like a fraction. If we want to compare two quantities we can divide both numbers, then we can express it:

- a) As a fraction.
- b) As a decimal number.
- c) As a ratio.

Example: Compare the numbers 6 and 2, we write $\frac{6}{2}=3$ which means that 6 contains the number 2 three times.

Comparing now the numbers 5 and 2: We can write the comparison as:

- a) A fraction: $\frac{5}{2}$.
- b) A decimal number 2,5.
- c) A ratio 5:2 (read “5 to 2”)

The three possibilities have the same meaning, the difference between a ratio and a fraction is that in fractions we only use whole numbers and in ratios we can also use decimal numbers.

As fractions and ratios mean the same thing, the properties of the fractions can also use in ratios, one important property is:

We can multiply and divide both terms in a ratio by the same number.

A ratio in its most simple form is the fraction in its lowest terms (always expressed with whole numbers).

Examples: These ratios are equivalent:

12:8=3:2 (3:2 is the ratio expressed in its simplest form).

4,5:9=9:18=1:2 (1:2 is the ratio expressed in its simplest form).

Your turn

1. Express the following ratios in their simplest form:

a) 16 to 8

b) 8 to 20

c) 4,5:5

d) 4,5:3

e) 3/7:2

f) 15:21

Proportions:

When two ratios are equal, the four terms are in **proportion** or are a proportion, so $\frac{3}{7} = \frac{6}{14}$ or $3:7=6:14$ (is read “three is to seven as six is to fourteen) are a proportion.

If we write $\frac{a}{b} = \frac{c}{d}$, b and c are called the **means**, and a and d are called the **extremes**.

A fundamental property of proportions is that **the product of the extremes is equal to the product of the means**; this is very useful when we know three of the numbers of a proportion and want to calculate the fourth.

If we have $\frac{a}{b} = \frac{x}{d}$, we say $b \cdot x = a \cdot d$ and then $x = \frac{a \cdot d}{b}$

The rule is **one mean is the product of the extremes divided by the other mean**.

If we have $\frac{a}{x} = \frac{c}{d}$ and $x = \frac{a \cdot d}{c}$

The rule is **one extreme is the product of the means divided by the other extreme**.

Your turn

1. Write three different proportions with these ratios:

a) 3:2

b) $\frac{7}{5}$

c) 2,3:5

d) 7,2:4

2. Calculate the unknown value in these proportions:

a) $\frac{2}{3} = \frac{5}{x}$

b) $\frac{5}{2,4} = \frac{x}{8}$

c) $\frac{x}{9} = \frac{12}{24}$

d) $\frac{2}{x} = \frac{9}{2}$

3. Which number has the same ratio to 5 as to 9?

4. Find x if $\frac{4}{12} = \frac{12}{x}$ (x is called third proportional to 4 and 12).

5. Find x if $\frac{18}{5} = \frac{7}{x}$ (x is the fourth proportional to 18, 5 and 7).

6. Find x if $\frac{4}{x} = \frac{x}{5}$.

Direct proportions:

We say that there is a direct proportionality between two magnitudes if an increase on one magnitude causes a proportional increase on the other magnitude, and a decrease on the first quantity causes a proportional decrease on the second one.

Note that a direct proportion is the same as proportions we have seen on the previous point.

A direct proportion is also called simple proportion.

The best way to recognize if two magnitudes are in direct proportion is to see if when double one the other also doubles and if we half the first, the other also halves.

When we are trying to find a number in an exercise involving magnitudes in direct proportion there two methods:

Method 1: Unitary method:

1. We convert the proportion in 1:n or n:1 (the most convenient).
2. We multiply by the third quantity.

Example: A man walks 5200 m in $2\frac{1}{2}$ hours. How much will he walk in 7 h at the same speed?

If he walks 5200 in 2,5 hours, in 1 hour he will walk $\frac{5200}{2,5}=2080$ metres.

In 7 hours he will walk $2080 \cdot 7=14560$ m.

Method 2: The fractional method or proportion:

You must work as we have seen before with proportions.

Example: With the same data as in the last example:

$$\left. \begin{array}{l} 5200 \text{ m} \longrightarrow 2,5 \text{ h} \\ x \text{ m} \longrightarrow 7 \text{ h} \end{array} \right\} \Rightarrow \frac{5200}{x} = \frac{2,5}{7} \Rightarrow x = \frac{5200 \cdot 7}{2,5} = 14560 \text{ m}$$

Your turn 

1. Complete the table with the cost in pounds of a piece of silk.

Metres of silk	3	7		12	1
Cost in £	5		9		

2. Complete the table if the magnitudes are directly proportional.

x	1		4	2	
y		5	50		7

3. Complete the table of the cost in € and the litres of petrol bought.

Petrol in litres	1		5	7	
Cost in €		40	6,2		50

4. A phone call costs € 0,25 each 2 minutes or fraction rounded to the seconds. Complete the table.

Call length in minutes	7,5		13 minutes 25 seconds
Cost in €		12	

5. Richard earns £17,5 for working 7 hours. How much will he earn for working 9 hours.

6. We have paid for 7 nights in “Hotel Los Llanos” 364 €. How much will we pay for 3 nights?
How much for 15 nights?

7. For cooking a cake for 6 people the recipe says that we need 3 eggs, 150 g of flour and 50 g of sugar. Calculate how much of each ingredient we need to cook a cake for 9 people.

8. Maria has exchanged \$ 370 and has received € 255 without any commission. How much in € will she receive for \$ 20? How much in \$ will she receive for € 60?

9. A 25 kg tin of paint covers 70 m^2 of wall. How many kg would be needed to cover 53 m^2 of wall?

14. My car uses 16 litres of petrol to travel 250 km.
 a) How far can I travel with 55 litres?
 b) How much petrol would I need to travel 180 km?

Inverse proportions:

We say that there is an inverse proportionality between two magnitudes if an increase on one magnitude causes a proportional decrease on the other magnitude, and a decrease on the first magnitude causes a proportional increase on the other one. This means that if one magnitude is multiplied by 2, 3, 4, ... the second magnitude is divided by 2, 3, 4, ...

Example: If 18 men can do a job in 10 days, in how many days will 45 men do the same job?

This is an inverse proportion because with the double of the men, half the days are required.

$$\left. \begin{array}{l} 18 \text{ men} \xrightarrow{\text{I}} 10 \text{ days} \\ 45 \text{ men} \xrightarrow{\quad} x \text{ days} \end{array} \right\} \Rightarrow \frac{18}{45} = \frac{x}{10} \Rightarrow x = \frac{18 \cdot 10}{45} = 4 \text{ days}$$

Your turn 

- Two pumps take 5 days to empty a pool. How long will 5 pumps take to empty the same pool?
- At 65 km/h a journey takes 5 h 25 min. How long will the journey take at a speed of 75 km/h?
- It takes 12 hours for 3 bricklayers to build a wall. How long will it take for 5 bricklayers?

4. At 180 km/h a train takes 1 h 23 min. for its journey from Albacete to Valencia. How long will the same journey take with the AVE at a speed of 190 km/h? How long with a speed of 250 km/h?

5. A company need 33 workers to pack a its production in 25 days. If the total production needs to be packed in 15 days, how many extra workers do they need?

6. James can write 8 pages with 25 lines per page in one hour. How many pages can he write if there are 20 lines on each page?

7. If 2 eggs take 6 minutes to boil, how long will 5 eggs take?

8. There have been 13 winners in a lottery and each one will receive 23000 €, but there are 3 more winners, how much will each one receive now?

9. If I ride my bicycle at an average speed of 15 km/h I travel a distance of 22 km in a certain period of time. If the speed is 17 km/h, how far will I travel?

Percentages:

A percent is a ratio to 100.

A percentage can be considered as a fraction with denominator 100.

Percent and hundredths are basically equivalent. This makes conversion between percent and decimals very easy.

To convert from a decimal to a percent, just move the decimal 2 places to the right.

To convert a fraction into a percentage we multiply the the numerator by 100 and we make the division.

There are some very easy cases of percentages that can match mentally to fractions such as $50\% =$

$$\frac{1}{2}, 25\% = \frac{1}{4}, 20\% = \frac{1}{5}, 10\% = \frac{1}{10}, \text{ etc.}$$

Examples:

a) The ratio 3:8 or the fraction $\frac{3}{8}$ can be expressed as $\frac{3 \cdot 100}{8} = 37,5\%$

b) $\frac{3}{5} \Rightarrow \frac{3 \cdot 100}{5} = 60\%$

Summarising ideas we can write:

As a proportion or a fraction	As a decimal number	As a percentage
3:8	0,375	37,5%
$\frac{3}{5}$	0,6	60%
Making the division →	Multiplying by 100 →	

Your turn 

Complete the table:

Proportion or fraction	Decimal number	Percentage
		30%
		70%

	0,27	
	1,5	
$\frac{2}{5}$		
$\frac{5}{4}$		
	0,68	
		28,50%
$\frac{5}{8}$		

Calculating a percentage of a quantity:

To calculate the percentage of a quantity we must multiply it by the percent and divide by 100.

Example: Calculate the 45% of 24:

$$45 \text{ of } 24 = \frac{45}{100} \cdot 24 = \frac{45 \cdot 24}{100} = 10,8$$

Your turn 

1. Calculate:

a) 25% of 28

b) 37,5% of 500

c) 70% of 45

d) 3% of 700

2. The population of a town is 652 000 and 35% of them live in the centre district. How many of them live in this district?

3. You have the possibility of choosing a prize:

Prize 1: 25% of 570€

Prize 2: 30% of 450€

Prize 3: 95% of 150€

Which one do you choose?

Calculate the total from the percent:

Calculating the total from the percent can be done using direct proportion.

Example: In the class 13 students didn't do their homework, this was 52% of the class. How many students are in the class?

PART	→	TOTAL
52	→	100
13	→	x

$$x = \frac{13 \cdot 100}{52} = 25 \text{ students}$$



1. In a sale the price of a television set is 150€ which is 65% of the usual price, what was the original price?
2. The 6% of the population of Albacete are immigrants and there are 9900 immigrants living in this city, what is the population of Albacete?

Percentage increase-decrease:**Calculate a number increased or decreased in a percentage:**

It can be done in two steps:

1. Calculate the % of the quantity.
2. Add or subtract the percentage to the original quantity.

Example: The population of a town was 63 500 last year, and it increased by 8% this year, what is the population now?

$$1^{\text{st}} \text{ step: } 8\% \text{ of } 63500 = \frac{8 \cdot 63500}{100} = 5080$$

$$2^{\text{nd}} \text{ step: The current population is } 63500 + 5080 = 68580$$

Finding the original amount:

If we know the % increase or decrease and the value we can find the total using proportions.

Example: The net salary of an employee is 1230e after paying 18% of IRPF, what is the gross of his/her salary?

82 € net \longrightarrow 100€ gross

1230 € net \longrightarrow x€ gross $x = \frac{1320 \cdot 100}{82} = 1500$



1. The price of some clothes is 68€ and there is a discount of 7%, what is the final price?

2. I have bought a pair of jeans for 32€, the IVA is 16%, what was the price before?

3. For each quantity including 12% IVA, calculate the original cost excluding IVA.

a) 147,84€

b) 65€

c) 2072€

d) 500€

4. The price of an electric oven before taxes is 560€ plus 17% IVA and the salesman offers a 12% discount, what is the final price?

5. Find the current price for each item:

Item	Car	Coat	Book	Bus ticket	Holiday	Globes	House
Old price	25.000 €	350€	58€	2,5€	1800€	25€	250.000€
Change	7% incr.	30% disc.	12% disc.	12% disc.	6% disc.	25 disc.	3% incr.

Finding the % increase or decrease:

If we know the amount of the increase or decrease and the initial value we can find the % increase or decrease using proportions.

Initial value —————> increase/decrease
 100 —————> x% increase/decrease

Example: Last year, the number of students of the bilingual section of a school was 20 and this year it is 35. What has been the % increase of the number of students?

20 students —————> 35 students

100 students —————> x $x = \frac{15 \cdot 100}{20} = 75 \%$

Your turn 

1. Last year there were 1560 employees in a company, this year 230 new people have been employed. What has been the % increase of the staff in the company?
2. I have paid 215 € for a coat and the original price was 230 €. What is the % discount?

Keywords:

Ratio= Razón

Proportion= Proporción

mean= medio

extreme= extremo

Direct proportionality= proporcionalidad directa

magnitude= magnitud

Inverse proportionality= proporcionalidad inversa

percentage= porcentaje

percent= por ciento

increase= incremento, aumento

decrease= disminución, descenso