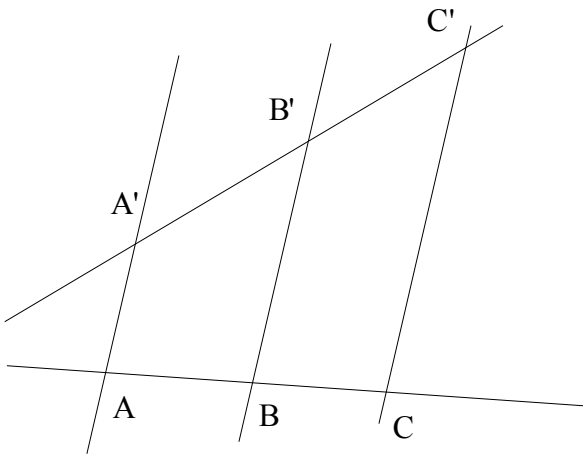
UNIT 9: GEOMETRIC PROPORTIONALITY. SIMILARITY.

Thales Theorem:

If two non parallel straight lines intersect with parallel straight lines, the ratios of any two segments on the first line are equal to the ratios of the corresponding segments on the second line.

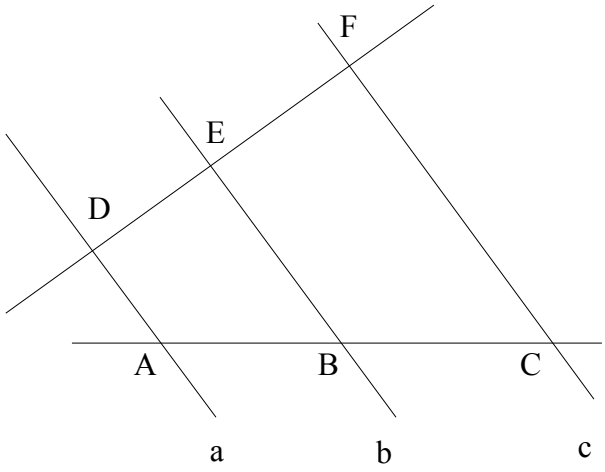


That is: $\frac{\overline{AB}}{\overline{BC}}=\frac{\overline{A'B'}}{\overline{B'C'}}$, you can also say that $\frac{\overline{AB}}{\overline{A'B'}}=\frac{\overline{BC}}{\overline{B'C'}}$, or even you can say:

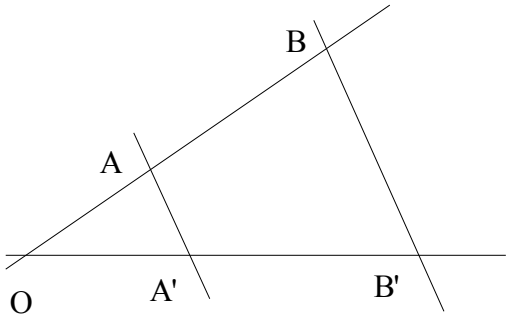
$$\frac{\overline{AB}}{\overline{A'B'}}=\frac{\overline{BC}}{\overline{B'C'}}=\frac{\overline{AC}}{\overline{A'C'}}$$

1. Consider that lines a, b and c are parallel lines and

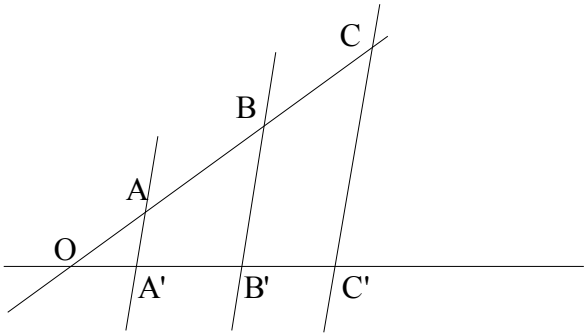
$\overline{AB}=2$ cm
$\overline{EF}=2{,}8$ cm and
$\overline{BC}=3{,}5$ cm
Calculate $\overline{DE}$



2. Calculate the length of the segment $\overline{A'B'}$, if you know $\overline{OA}=3$ cm, $\overline{AB}=6$ cm and $\overline{OA'}=4$ cm.



3. Calculate the length of the segments $\overline{OA'}$ and $\overline{BC}$:

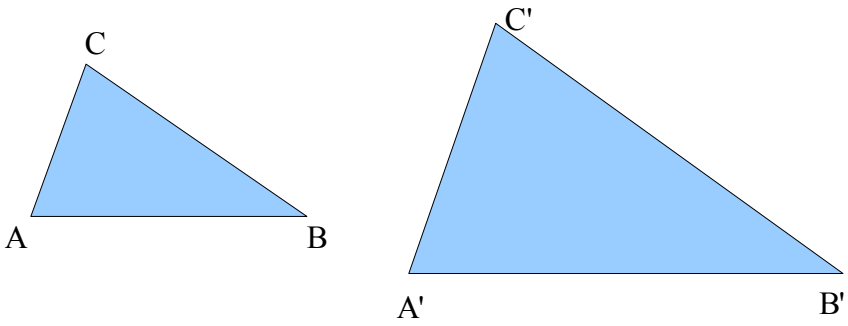


$\overline{OA}=7$ cm ; $\overline{AB}=10$ cm ; $\overline{A'B'}=8$ cm ; $\overline{B'C'}=6$ cm

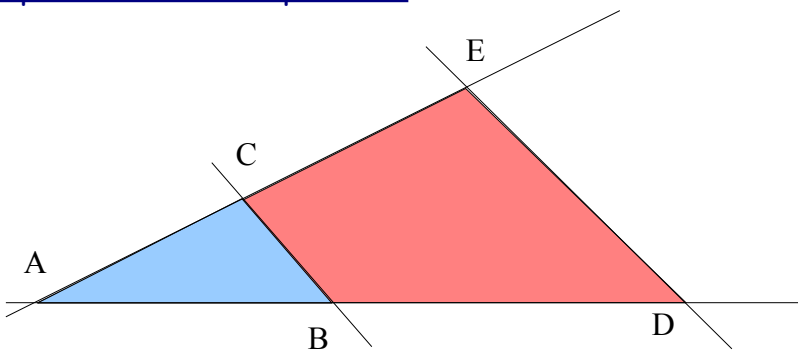
Similar Triangles:

Two triangles ABC and A'B'C' are similar if:

- Their angles are equal: $\hat{A}=\hat{A}'$; $\hat{B}=\hat{B}'$; $\hat{C}=\hat{C}'$
- Their sides are proportional: $\frac{\overline{AB}}{\overline{A'B'}}=\frac{\overline{BC}}{\overline{B'C'}}=\frac{\overline{AC}}{\overline{A'C'}}$.

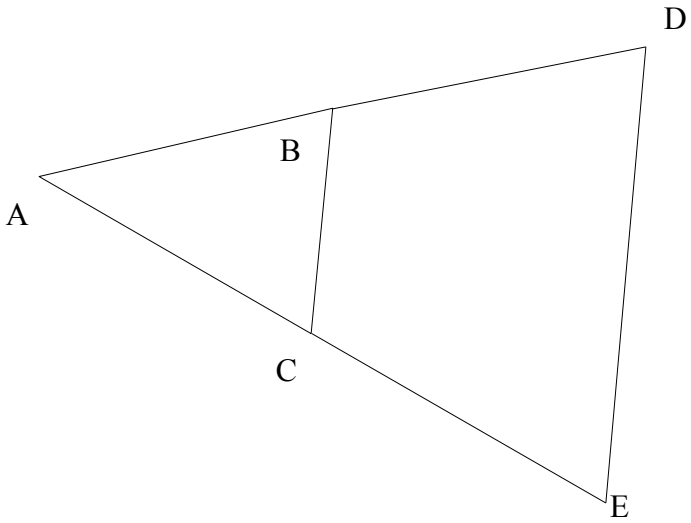


Triangles put in the Thales position:



If we use the Thales Theorem in the drawing above in which the non parallel lines intercept at A, the triangle ABC is inside the triangle ADE, we say that these two triangles are in Thales position and they are similar because corresponding angles are equal and corresponding sides are in proportion.

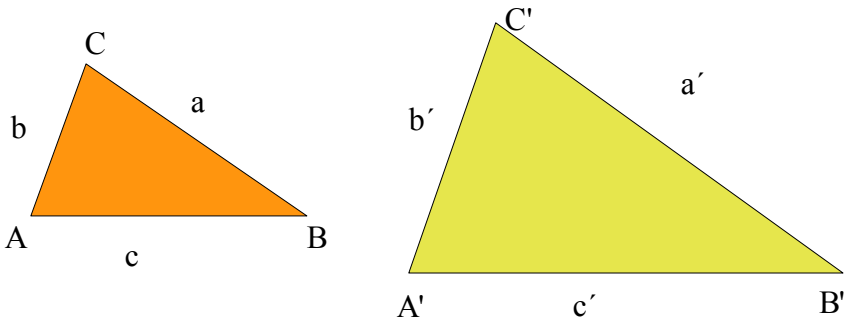
1. Use the Thales theorem to find DE in the picture below:



$\overline{AB} = 6{,}2$ cm ; $\overline{AD} = 10{,}5$ cm ; $\overline{BC} = 4{,}86$ cm

Similarity Criteria of Triangles:

To be sure that two triangles are similar, we do not need to see if all the three corresponding angles are equal and corresponding sides are in proportion. We can assure that they are similar with fewer conditions. These sets of conditions are called similarity criteria.



FIRST SIMILARITY CRITERIUM: Two triangles ABC and A'B'C' are similar if all the three sides are in proportion, that is $\frac{a}{a'}=\frac{b}{b'}=\frac{c}{c'}$.

(This is true because these triangles could be put in Thales position).

SECOND SIMILARITY CRITERIUM: Two triangles ABC and A'B'C' are similar if $\hat{A}=\hat{A}'$ and $\hat{B}=\hat{B}'$.

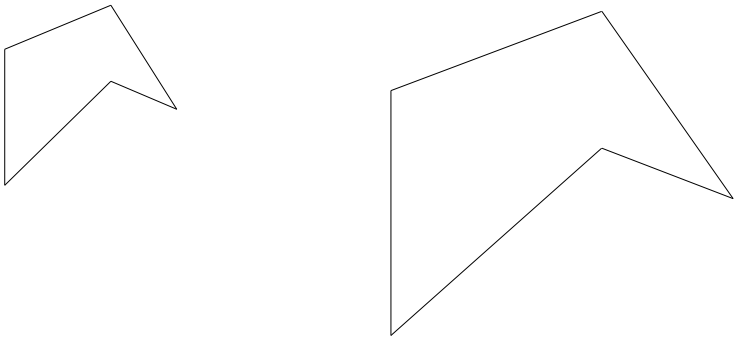
(This is true because in this case we know $\hat{C}=\hat{C}'$. Remember that $\hat{A}+\hat{B}+\hat{C}=\hat{A}'+\hat{B}'+\hat{C}'=180^{\circ}$).

THIRD SIMILARITY CRITERIUM: Two triangles are similar if $\hat{A}=\hat{A}'$ and $\frac{b}{b'}=\frac{c}{c'}$.

(This is true because the two triangles could be put in the Thales position).

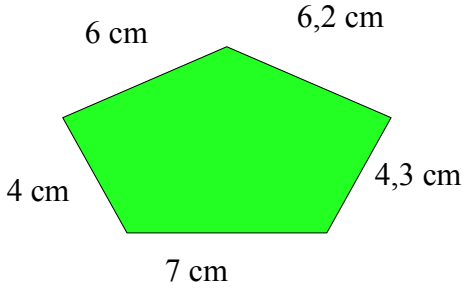
Similar Polygons:

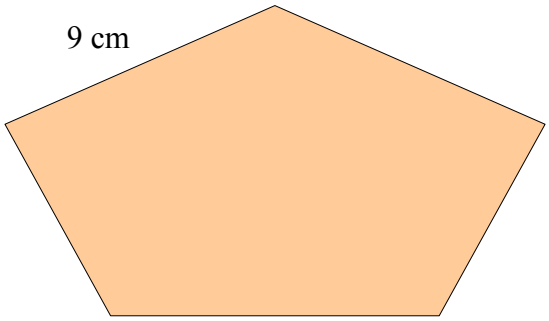
Two polygons are similar if their angles are equal and their sides are in proportion.



The **similarity ratio** is the quotient between the length of a side of a polygon and the length of the corresponding side of the other polygon.

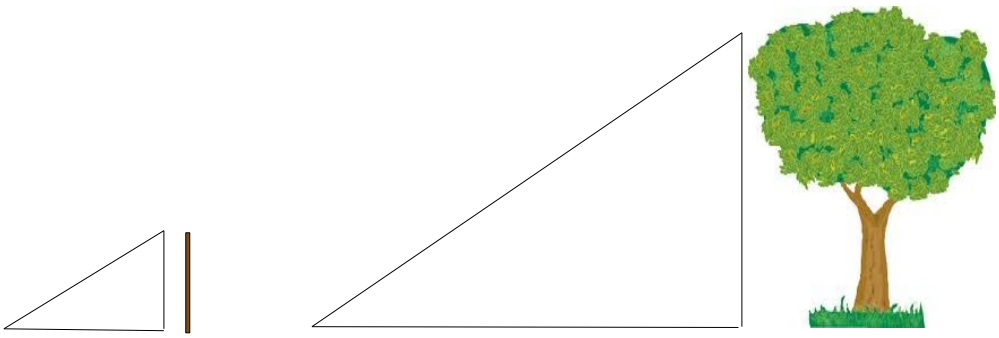
1. Calculate the unknown sides of these similar shapes :





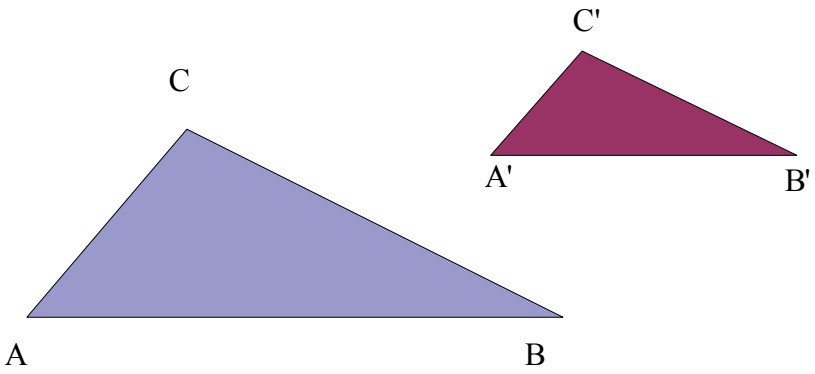
2. The three angles of a triangle ABC are a=2 cm, b=2,7 cm and c= 3,4 cm. The triangle A'B'C' is similar and a'=2,5 cm. Which are the values of the other two sides?

3. Measuring the length of the shadow of a stick, we can calculate the height of a tree. Calculate the height of the tree from the picture below considering that the length of the stick is 1,25 m, its shadow is 1,52 m and the shadow of the tree is 6,3 m.

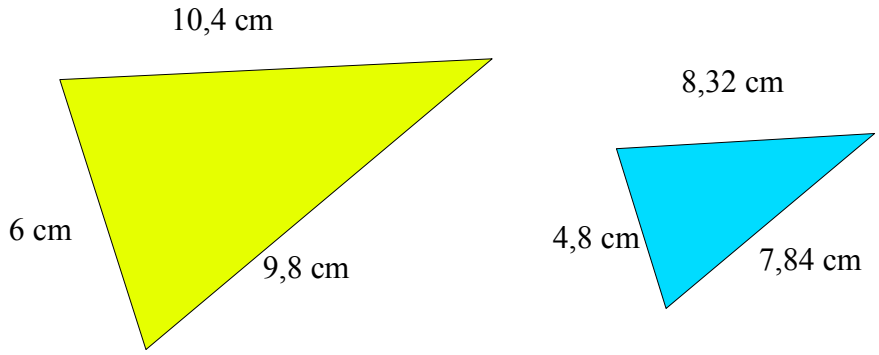


4. The triangles ABC and A'B'C' have $\hat{A}=\hat{A}'=43{,}7^{o}$ and $\hat{C}=\hat{C}'=109{,}3^{o}$.

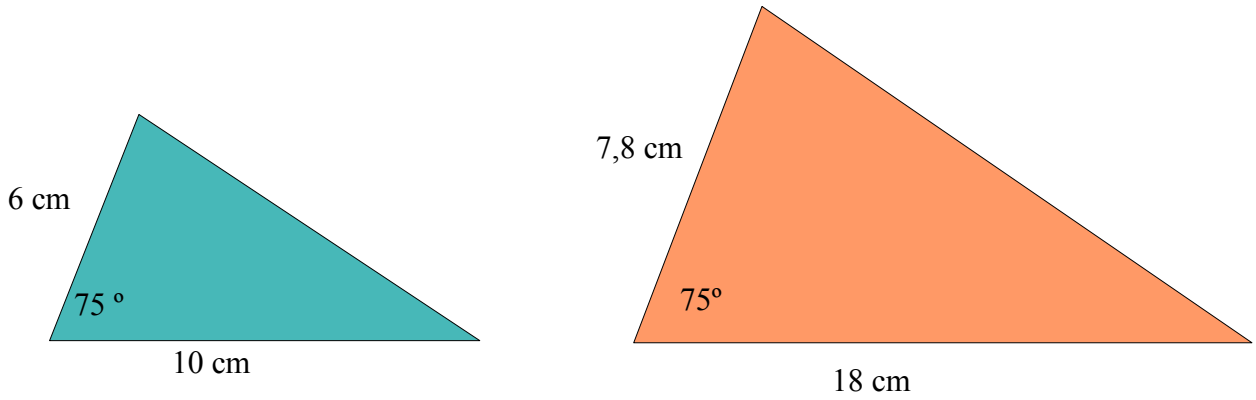
a) Check that they are similar triangles.
b) Find the value of $\hat{B}$.
c) Calculate the similarity proportion if you know: a=7 cm and a'=4,2.



5. Prove without measuring that these triangles are similar. Calculate the similarity ratio.



6. Prove without measuring that these triangles are similar. Calculate the similarity ratio.



Keywords:

Thales Theorem=Teorema de Thales
straight lines=lineas rectas, rectas
parallel straight lines=rectas paralelas
segment=segmento
similarity=semajanza
similar triangles=triángulos semejantes
similarity ratio=razón de semejanza
angle=ángulo
side=lado
similarity criteria of triangles=criterios de semejanza de triángulos
similar polygons=polígonos semejantes