

UNIT 3: DECIMAL NUMBERS:

Decimal Numbers:

Remember that to express numbers that are not whole numbers we use decimal numbers as 64,295 in which every digit has a value which is divided by ten when we move to the right. So

6 is sixty units

4 is four units

2 is two tenths of the unit

9 is nine hundredths of the unit

5 is five thousandths of the unit

And we continue like that if there are more digits.

This is the decimal system that is commonly use nowadays except, sometimes for time and angles.

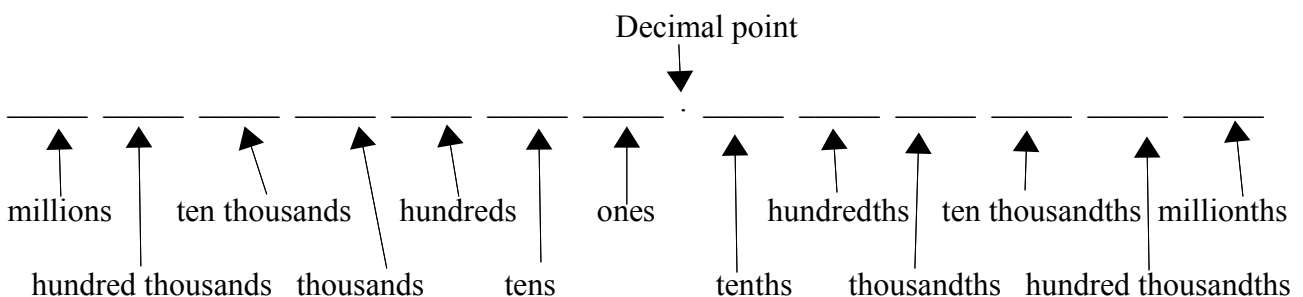
A decimal number has two parts:

2,58 2 is the **whole number part**
 58 is the **decimal part.**

You say: two point five eight (Or two units and fifty eight hundredths).

(When we write the number in Spanish we use a comma, but remember that English speakers use a dot, the **decimal point**).

You can use a **decimal value diagram** to see the value of each digit. Since decimal is a base 10 number, each place has a value that is either ten times larger than the one on its right, or ten times smaller than the one on its left.





1. Draw a line to match each number to its equivalent value in words:

0.324	Fourteen point nine six
106.7	Thirty-two units and four tenths
14.96	One hundred and six point seven
5.09	Five hundred and nine
32.4	Three hundred and twenty-four thousands
509	Five units and nine hundredths

2. Write each number as digits, putting in decimal points where needed:

- a) One hundred and sixty five point two four.
- b) Fifty-seven units and ninety-two hundredths.
- c) One hundred and one point three.
- d) Ten units and two tenths.
- e) Zero point five six.
- f) Ninety four thousandths.
- g) One and three ten thousandths.
- h) Six millionths.

3. Give the value of the digit 8 in each of these numbers:

38 002 _____
18 005 407 _____
23 804 000 _____
45,876 _____
4,0871 _____

4. Write in words:

3,006
45,204
\$3,45
34,40 metres

Types of decimal numbers:

There are three different types of decimal numbers:

- An **exact or terminating** number is one which does not go on forever, so you can write down all its digits. For example: 0,125
- A **recurring or repeating** decimal is a decimal number which does go on forever, but where some of the digits are repeated over and over again. For example: $0,125252525\dots = 0,1\overline{25}$
We can distinguish:
 - Decimals that the repeating part or period starts just after the decimal point (**pure recurring decimal**). For example: $0,323232\dots = 0,\overline{32}$
 - Decimals that the repeating part or period does not start just after the decimal point (**mixed recurring decimal**). For example: $0,10232323\dots = 0,10\overline{23}$
- Other decimals (**Irrational** numbers, they cannot be written as fractions) are those which go on forever and don't have repeated digits. For example:
 $\sqrt{2} = 1,4142135\dots$

Decimal expression of a fraction:

To calculate the decimal expression of a fraction we divide the numerator by the denominator. We can have the following expressions:

- An integer, if the numerator is a multiple of the denominator.
- An exact number, if after the simplification of the fraction the numerator only has as factors either 2 or 5.
- A repeating decimal, if after the simplification of the fraction, the numerator has different factors of 2 and 5.

Examples:

$$\frac{12}{3} \xrightarrow{\text{12 is a multiple of 3}} \text{Integer} \qquad \frac{12}{3} = 4$$

$$\frac{57}{30} = \frac{19}{10} \xrightarrow{\text{10=2·5 (only factors 2 and 5)}} \text{Exact Decimal} \qquad \frac{57}{30} = 1,9$$

$$\frac{7}{9} \xrightarrow{\text{9=3}^2 \text{ (different factors of 2 and 5)}} \text{Repeating Decimal} \qquad \frac{7}{9} = 0,777\dots = 0,\hat{7}$$

Ordering decimal numbers:



1. The number 9,162 is bigger than 9,17. True or false?
2. The number 0,175 is bigger than 0,2. True or false?
3. Which is the largest: 96,145 ; 96,2 or 96,19?
4. Which is the smallest: 25,62 ; 96,2 or 25,6?
5. Put these number in order, starting with the largest:
75,1 25,7 75,06 25,008
6. Put these numbers in order, starting with the smallest:
0,17 0,8 0,072 0,06 0,064

Operations with decimal:

Addition and Subtraction:



1. Calculate:

a) $79,82 + 7,091 - 12,1274$

b) $(9,03 - 1,5) - (3,002 + 1,13)$

c) $(1,91 - 0,003) + (10 - 4,756)$

d) $12,879 - (13,4 + 1,51 - 9,03)$

Multiplication:



1. Calculate:

a) $1,6 \cdot 0,27$

b) $(0,92 - 0,004) \cdot 2,5$

c) $12,3 \cdot (5,6 + 1,06)$

d) $23,76 - 3,04 \cdot 2,9$

Division:

1. Calculate:

a) $92,14 : 5$

b) $760,6 : 12$

c) $17,38 : 1,4$

d) $29,34 : 0,006$

Square root:**Calculate square root without using a calculator:****Example:** Find $\sqrt{645}$ with one decimal place:

First group the numbers under the root in pairs from right to left, leaving either one or two digits on the left (6 in this case). For each pairs of numbers you will get one digit in the square root.

To start, find a number whose square is less or equal to the first pair or first number, and write it next to square root symbol:

$\begin{array}{r} \sqrt{6\widehat{4}5} \\ -4 \\ \hline 245 \end{array}$	$\begin{array}{r} 2 \\ \hline \end{array}$
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Square the 2, giving 4, write that underneath 6, and subtract. Bring down the next pair of digits.

$$\begin{array}{r|l} \sqrt{64\widehat{5}} & \underline{2} \\ -4 & \underline{4} \\ \hline 245 & \end{array}$$

Then double the number 2 (highlighted), and write it down below.

$$\begin{array}{r|l} \sqrt{64\widehat{5}} & \underline{2} \\ -4 & \underline{45 \cdot 5 = 225} \\ \hline 245 & \\ -225 & \\ \hline 20 & \end{array}$$

We don't consider the last digit of 245, so we get 24, and divide by 4, multiplying $45 \cdot 5$, we get 225 and we subtract.
 $45 \cdot 5 = 225$; $245 - 225 = 20$

$$\begin{array}{r|l} \sqrt{64\widehat{5}} & \underline{25} \\ -4 & \underline{45 \cdot 5 = 225} \\ \hline 245 & \\ -225 & \\ \hline 20\ 00 & \end{array}$$

Write 5 on the top, and bring down the next pair of digits (in this case the decimal digits 00)

$$\begin{array}{r|l} \sqrt{64\widehat{5}} & \underline{25.3} \\ -4 & \underline{45 \cdot 5 = 225} \\ \hline 245 & \\ -225 & \underline{503 \cdot 3 = 1509} \\ \hline 2000 & \\ -1509 & \\ \hline 491 & \end{array}$$

Then double 25, and do the same than before.
 Don't forget to write the decimal point after 25.

Thus, with one decimal place, $\sqrt{645} = 25,3$



1. Calculate the square root of the following numbers with a decimal place:

a) 268

b) 162

c) 2625

d) 3780

Rounding decimal numbers:

Remember that to round a number to any decimal place value we look at the digit to the right of the place we wish to round to and when the digit 5, 6, 7, 8 or 9 appears in that place, you must add one unit to the last digit; when the digit 0, 1, 2, 3 or 4 appears in that place, you must simply cut the number.

Example:

1,17 rounded to the nearest tenth is 1,2 while 3,562 rounded to the nearest hundredth is 3,56.

Activities.



1. Round off the numbers:

2,36 to the nearest tenth

6,3757 to the nearest hundredth

0,8903 to the nearest thousandth

17,17 to the nearest unit

176,705 to the nearest hundred

67,756 to the nearest thousand

2. We have paid €5,88 for a portion of cheese. What does it weigh if the whole cheese costs € 12,25 a kilo?

3. 1,6 kg of cherries costs € 6. How much does a kilo of cherries cost?

3. Frank asks the butcher for three steaks that, once cut, weigh 708 grams total. How much must he pay if one kilo of steaks costs € 9,35?

4. Julian is 13 years old and 1,72 metres tall. When he was 8 years old he was 1,57 metres tall. What is his average growth per year?

5. How much does it cost a family to go to the zoo if there are two parents, two children and a grandfather?
TICKET PRICES: ADULTS: €5,35 CHILDREN: €2,80

6. Robert goes to the market with € 62,81 and buys 2,6 kg of grapes for 1,80 €/kg, 0,58 kg of bananas for 2,15 €/kg, some hake that weighs 850 g and costs 11,45 €/kg, and a chicken that weighs 1,15 kg and costs 5,59 €/kg. How much money is left over?

7. We want to paint a fence that is 147,8 m long and 1,8 m high. A kilo of paint costs € 7,35 and covers 1,20 square metres. Calculate the estimated cost of the pain.

Keywords:

Decimal number= **Número decimal**

whole number part= **parte entera**

decimal part= **parte decimal**

units or ones= **unidades**

tens= **decenas**

hundreds= **centenas**

thousands= **unidades de mil (millar)**

ten thousands= **decenas de mil (millar)**

hundred thousands= **centenas de mil (millar)**

millions= **unidades de millón**

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tenths= **décimas**

hundredths= **centésimas**

thousandths= **milésimas**

ten thousandths= **diesmilésimas**

hundred thousandths= **cienmilésimas**

millionths= **millonésimas**

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decimal point= **punto para el decimal (nosotros usamos coma)**

exact or terminating decimal number= **número decimal exacto**

recurring or repeating decimal number= **número decimal periódico**

pure recurring decimal= **decimal periódico puro**

mixed recurring decimal= **decimal periódico mixto**

Rational Number= **Número Racional**

Irrational Number= **Número Irracional**

to round off a number= **redondear un número**